

Math 62 13.1-3rd Review of Sequences & Series

Math 72 11.3-2nd

Review: Sequences and Series

Arithmetic sequence:

- Add (or subtract = add negative) the same number d each time
- $a_n = a_1 + d(n - 1)$

Geometric sequence

- Multiply (or divide = multiply by a fraction) the same number r each time
- $a_n = a_1 \cdot r^{n-1}$

Practice

Write the general term for each sequence.

1) $2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}$

2) $2, \frac{8}{3}, \frac{10}{3}, 4, \frac{14}{3}$

3) $6, -3, \frac{3}{2}, -\frac{3}{4}, \frac{3}{8}$

4) $2, -1, -4, -7, -10$

Write in summation notation.

5) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$

6) $2 + \frac{8}{3} + \frac{10}{3} + 4 + \frac{14}{3} + \frac{16}{3}$

7) $6 - 3 + \frac{3}{2} - \frac{3}{4} + \frac{3}{8}$

8) $2 - 1 - 4 - 7 - 10 - 13 - 16 - 19 - 21$

9) Find a_{100} when $a_n = \frac{(-1)^{n+1} \cdot n^2}{4}$

10) Evaluate

$$\sum_{n=1}^6 3 \cdot 2^n$$

11) Evaluate S_3 when $a_n = 7n - 1$

12) Write the general term: $3, -2, -7, -12, -17$

13) Write in summation notation: $10 + 2 + \frac{2}{5} + \frac{2}{25}$

14) Find the first six terms of $a_n = 7 \left(-\frac{2}{5}\right)^{n+1}$

Review: Sequences and Series

Write the general term:

① $2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}$

↪

$\times \frac{1}{3}$ mult each time $\Rightarrow$ geometric

$$a_1 = 2$$

$$r = \frac{1}{3}$$

$$a_n = a \cdot r^{n-1}$$

$$a_n = 2 \cdot \left(\frac{1}{3}\right)^{n-1}$$

or

$$a_n = \frac{2}{3^{n-1}}$$

② $2, \frac{8}{3}, \frac{10}{3}, 4, \frac{14}{3}$

rewrite with common denominator

$$\frac{6}{3}, \frac{8}{3}, \frac{10}{3}, \frac{12}{3}, \frac{14}{3}$$

↪

$+\frac{2}{3}$ add each time $\Rightarrow$ arithmetic

$$a_1 = 2$$

$$d = \frac{2}{3}$$

$$a_n = a_1 + d(n-1)$$

$$a_n = 2 + \frac{2}{3}(n-1)$$

$$= 2 + \frac{2}{3}n - \frac{2}{3}$$

$$a_n = \frac{2}{3}n + \frac{4}{3}$$

③ $6, -3, \frac{3}{2}, -\frac{3}{4}, \frac{3}{8}$

↪

$\times (-\frac{1}{2})$ mult each time $\Rightarrow$ geometric

$$a_1 = 6$$

$$r = -\frac{1}{2}$$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_n = 6 \left(-\frac{1}{2}\right)^{n-1}$$

or

$$a_n = \frac{6}{(-2)^{n-1}}$$

④ $2, -1, -4, -7, -10$

↙
-3 subtract each time $\Rightarrow$ arithmetic

$$a_1 = 2$$

$$d = -3$$

$$a_n = a_1 + d(n-1)$$

$$= 2 + (-3)(n-1)$$

$$= 2 - 3n + 3$$

$$\boxed{a_n = -3n + 5}$$

Write in summation notation

⑤ $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$

4 terms

$$\sum_{i=1}^4 a_i$$

a_i = general term

Note: This question matches ①

$$= \boxed{\sum_{i=1}^4 2\left(\frac{1}{3}\right)^{i-1}}$$

or $\boxed{\sum_{n=1}^4 2\left(\frac{1}{3}\right)^{n-1}}$

⑥ $2 + \frac{8}{3} + \frac{10}{3} + 4 + \frac{14}{3} + \frac{16}{3}$

6 terms

$$\boxed{\sum_{n=1}^6 \left(\frac{2}{3}n + \frac{4}{3}\right)}$$

Note: This question matches ②

⑦ $6 - 3 + \frac{3}{2} - \frac{3}{4} + \frac{3}{8}$

5 terms

$$\boxed{\sum_{n=1}^5 6\left(\frac{-1}{2}\right)^{n-1}}$$

Note: This question matches ③

⑧ $2 - 1 - 4 - 7 - 10 - 13 - 16 - 19 - 21$

9 terms

$$\sum_{n=1}^9 (-3n+5)$$

Note: This question matches ④.

⑨ Find a_{100} when $a_n = \frac{(-1)^{n+1} \cdot n^2}{4}$

subst $n=100$ $a_{100} = \frac{(-1)^{100+1} \cdot (100)^2}{4}$

$$= \frac{(-1)^{101} \cdot 10000}{4}$$

odd power of (-1)
is -1

$$= -1 \cdot 2500$$

$$= \boxed{-2500}$$

⑩ Evaluate $\sum_{n=1}^6 3 \cdot 2^n$

$$\begin{aligned} & \begin{matrix} n=1 & n=2 & n=3 & n=4 & n=5 & n=6 \end{matrix} \\ & = 3 \cdot 2^1 + 3 \cdot 2^2 + 3 \cdot 2^3 + 3 \cdot 2^4 + 3 \cdot 2^5 + 3 \cdot 2^6 \\ & = 6 + 12 + 24 + 48 + 96 + 192 \\ & = \boxed{378} \end{aligned}$$

⑪ Evaluate S_3 when $a_n = 7n - 1$

$$S_3 = \sum_{n=1}^3 (7n-1)$$

$$\begin{aligned} & \begin{matrix} n=1 & n=2 & n=3 \end{matrix} \\ & = 7(1)-1 + 7(2)-1 + 7(3)-1 \\ & = 6 + 13 + 20 \\ & = \boxed{39} \end{aligned}$$

⑫ Write the general term 3, -2, -7, -12, -17

↪

-5 subtract 5 each time

arithmetic

$$a_1 = 3$$

$$d = -5$$

$$a_n = a_1 + d(n-1)$$

$$= 3 - 5(n-1)$$

$$= 3 - 5n + 5$$

$$= \boxed{-5n + 8}$$

⑬ Write in summation notation. $10 + 2 + \frac{2}{5} + \frac{2}{25}$

↪
 $\times \frac{1}{5}$

• need general term

mult by $\frac{1}{5}$ each time $\Rightarrow$ geometric

$$a_1 = 10$$

$$r = \frac{1}{5}$$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_n = 10 \left(\frac{1}{5}\right)^{n-1}$$

• 4 terms added

$$\boxed{\sum_{n=1}^4 10 \left(\frac{1}{5}\right)^{n-1}}$$

⑭ Find the first six terms of $a_n = 7 \left(-\frac{2}{5}\right)^{n+1}$

$$n=1$$

$$n=2$$

$$n=3$$

$$n=4$$

$$n=5$$

$$n=6$$

$$7 \left(-\frac{2}{5}\right)^{1+1}$$

$$7 \left(-\frac{2}{5}\right)^{2+1}$$

$$7 \left(-\frac{2}{5}\right)^{3+1}$$

$$7 \left(-\frac{2}{5}\right)^{4+1}$$

$$7 \left(-\frac{2}{5}\right)^{5+1}$$

$$7 \left(-\frac{2}{5}\right)^{6+1}$$

$$7 \left(-\frac{2}{5}\right)^2$$

$$7 \left(-\frac{2}{5}\right)^3$$

$$7 \left(-\frac{2}{5}\right)^4$$

$$7 \left(-\frac{2}{5}\right)^5$$

$$7 \left(-\frac{2}{5}\right)^6$$

$$7 \left(-\frac{2}{5}\right)^7$$

$$7 \left(\frac{4}{25}\right)$$

$$7 \left(\frac{-8}{125}\right)$$

$$7 \left(\frac{16}{625}\right)$$

$$7 \left(\frac{-32}{3125}\right)$$

$$7 \left(\frac{64}{15625}\right)$$

$$7 \left(\frac{-128}{78125}\right)$$

$$\boxed{\frac{28}{25}, \frac{-56}{125}, \frac{112}{625}, \frac{-224}{3125}, \frac{448}{15625}, \frac{-896}{78125}}$$